Nonlinear Relations between QCD Sum Rules for $\eta\Sigma\Sigma$, $\eta\Lambda\Lambda$ and $\pi\Lambda\Sigma$ Coupling Constants

A.Özpineci, S.B. Yakovlev, and V.S. Zamiralov

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M.V. Lomonosov Moscow State University

D.V. Skobeltsyn Institute of Nuclear Physics

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A.Özpineci [1] , S.B. Yakovlev, and V.S. Zamiralov

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[1]INFN, Sezione di Bari, Bari, Italy

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A.Özpineci
INFN, Sezione di Bari, Bari, Italy
email: Altug.Ozpineci@ba.infn.it

S.B. Yakovlev, and V.S. Zamiralov
D.V. Skobeltsyn Institute of Nuclear Physics, Moscow State University, Moscow, Russia
e-mail: stas@ix1.sinp.msu.ru
e-mail: zamir@depni.npi.msu.ru

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Summary

New relations between QCD Borel sum rules for strong coupling constants are derived. It is shown that starting from the sum rules for the coupling constants $g_{\mathcal{M}\Sigma\Sigma}$, $\mathcal{M}=\pi^0,\eta$, it is straightforward to obtain the corresponding sum rules for the $g_{\pi\Lambda\Sigma}$ and $g_{\eta\Lambda\Lambda}$ ones. In the given range of parameters the values of these couplings are obtained.

Резюме

Получены новые соотношения между борелевскими правилами сумм в КХД для констант сильной связи Σ^0 и Λ гиперонов. Показано, что, отправляясь от правил сумм для $g_{\mathcal{M}\Sigma\Sigma}$, $\mathcal{M}=\pi^0,\eta$, можно непосредственно получить соответствующие правила сумм для констант $g_{\pi\Lambda\Sigma}$ и $g_{\eta\Lambda\Lambda}$. В заданной области параметров вычислены значения этих констант связи.

1 Introduction

QCD sum rules [?] has been used thourouphly for studying the properties of the low-lying hadrons. Many attention has been paid to the analysis of the baryon magnetic moments to begin with [?]-[?] in the framework of the Borel sum rules. Analysis of the meson-baryon couplings in the framework of the QCD Borel sum rules are also of great interest (see [?]-[?] as these couplings enter many physical problems.

Recently we have found nonlinear relations between QCD sum rules for Σ^0 and Λ hyperons constructed for such important characteristics as masses and magnetic moments [?], [?]. These relations show that it is sufficient to construct a sum rule either for Σ^0 hyperon or for Λ one and to get all other sum rules just by definite nonlinear transformation.

It is natural to put a question whether similar relations can be constructed for other quantities such as pseudoscalar meson-baryon coupling constants. As we have noted in [?] the problem has also a practical side as for example QCD sum rules in [?],[?],[?],[?] were written for all πBB and ηBB coupling constants but the $\pi\Sigma\Lambda$ and $\eta\Lambda\Lambda$ ones.

We shall show here in what way one can construct QCD sum rules for the $\pi\Lambda\Sigma$ and $\eta\Lambda\Lambda$ couplings from those for $\pi\Sigma\Sigma$ and $\eta\Sigma\Sigma$ couplings on example of the known QCD sum rules derived in [?],[?]. Partly we have treated the problem in [?].

The paper is organized as follows. First we discuss relations between couplings of π^0 and η mesons to Σ and Λ hyperons in the SU(3) and quark model. Then we formulate relations between QCD correlators for Σ and Λ hyperons. In the next two sections we apply it to find QCD sum rules for $\pi\Lambda\Sigma$ and $\eta\Lambda\Lambda$ starting from the sum rule for π,η coupling to Σ^0 hyperon. In the last section results of the numerical analysis are given and discussed.

2 Relations between couplings of π^0 and η mesons to Σ and Λ hyperons in the SU(3)

We begin as in [?], [?] from a simple example. In the unitary model all the pseudoscalar meson-baryon coupling constants can be expressed in terms of F and D constants from the known unitary symmetry Lagrangian [?]

$$L = DSp\bar{B}\{P,B\} + FSp\bar{B}[P,B]. \tag{1}$$

where

$$B^\alpha_\beta = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^0 \end{pmatrix}. \tag{2}$$

$$P^\alpha_\beta = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}. \tag{3}$$

wherefrom

$$g_{\pi^0 pp} = \frac{1}{\sqrt{2}}(F+D), \quad g_{\pi^0\Sigma^+\Sigma^+} = \sqrt{2}F,$$

$$g_{\pi^0\Xi^0\Xi^0} = \frac{1}{\sqrt{2}}(F-D), \quad g_{\pi^0\Sigma\Lambda} = \sqrt{\frac{2}{3}}D,$$
$$g_{\eta pp} = \frac{1}{\sqrt{6}}(3F-D), \quad g_{\eta\Sigma^+\Sigma^+} = \sqrt{\frac{2}{3}}D,$$
$$g_{\eta\Xi^0\Xi^0} = -\frac{1}{\sqrt{6}}(3F+D), \quad g_{\eta\Lambda\Lambda} = -\sqrt{\frac{2}{3}}D.$$
$$g_{\pi pn} = (F+D), \quad g_{\eta\Sigma^0\Sigma^0} = \sqrt{\frac{2}{3}}D. \tag{4}$$

But for Σ-like baryons the coupling constants for π^0 and η mesons can be written in a form similar to that found for the corresponding baryon magnetic moments in $SU(3)_f$ [?],[?]:

$$\mu(B(qq', h) = (e_q + e_{q'})\mu_F + e_h(\mu_F - \mu_D),$$

wherefrom usual unitary symmetry pattern for the magnetic moments of the Σ-like baryon emerges:

$$\mu(p) = \mu(\Sigma^+) = \mu_F + \frac{1}{3}\mu_D,$$
$$\mu(\Xi^-) = \mu(\Sigma^-) = -\mu_F + \frac{1}{3}\mu_D,$$
$$\mu(n) = \mu(\Xi^0) = -\frac{2}{3}\mu_D.$$

Namely, let us write coupling constants of π^0 and η mesons related in the quark model to currents

$$j^{\pi^0} = \frac{1}{\sqrt{2}}[\bar{u}\gamma_5 u - \bar{d}\gamma_5 d] \tag{5}$$

and

$$j^{\eta} = \frac{1}{\sqrt{6}}[\bar{u}\gamma_5 u + \bar{d}\gamma_5 d - 2\bar{s}\gamma_5 s]. \tag{6}$$

with octet Σ-like baryons as $B(qq, h)$

$$g_{\mathcal{M}B(qq,h)B(qq,h)} = g_{\mathcal{M}qq}2F + g_{\mathcal{M}hh}(F-D), \tag{7}$$

or, particle per particle:

$$g_{\pi^0 pp} = g_{\pi^0 uu}2F + g_{\pi^0 dd}(F-D) = \sqrt{\frac{1}{2}}(F+D);$$
$$g_{\pi^0\Sigma^+\Sigma^+} = g_{\pi^0 uu}2F + g_{\pi^0 ss}(F-D) = \sqrt{2}F;$$
$$g_{\pi^0\Xi^0\Xi^0} = g_{\pi^0 ss}2F + g_{\pi^0 uu}(F-D) = \sqrt{\frac{1}{2}}(F-D);$$

and so on, where $g_{\pi^0 uu} = +\sqrt{\frac{1}{2}}$, $g_{\pi^0 dd} = -\sqrt{\frac{1}{2}}$ and $g_{\pi^0 ss} = 0$ can be just read off Eq.(??); and

$$g_{\eta pp} = g_{\eta uu}2F + g_{\eta dd}(F-D) = \sqrt{\frac{1}{6}}(3F-D);$$

$$g_{\eta\Sigma^+\Sigma^+} = g_{\eta uu}2F + g_{\eta ss}(F-D) = \sqrt{\frac{2}{3}}D;$$

$$g_{\eta\Xi^0\Xi^0} = g_{\eta ss}2F + g_{\eta uu}(F-D) = -\sqrt{\frac{1}{6}}(3F+D);$$

and so on, where $g_{\eta uu} = +\sqrt{\frac{1}{6}}$, $g_{\eta dd} = +\sqrt{\frac{1}{6}}$ and $g_{\eta ss} = -\sqrt{\frac{2}{3}}$ are read off the Eq.(??).

Let us write now a formal relations for the $\pi^0\Sigma^0\Sigma^0$ and $\eta\Sigma^0\Sigma^0$ coupling constants:

$$g_{\pi\Sigma^0\Sigma^0} = g_{\pi^0 uu}F + g_{\pi^0 dd}F + g_{\pi^0 ss}(F-D) = 0, \tag{8}$$

$$g_{\eta\Sigma^0\Sigma^0} = g_{\eta uu}F + g_{\eta dd}F + g_{\eta ss}(F-D) = \sqrt{\frac{2}{3}}D, \tag{9}$$

and change $(d \leftrightarrow s)$ to form an auxiliary quantities

$$g_{\pi^0\tilde{\Sigma}^0_{ds}\tilde{\Sigma}^0_{ds}} = g_{\pi^0 uu}F + g_{\pi^0 ss}F + g_{\pi^0 dd}(F-D) = \sqrt{\frac{1}{2}}D, \tag{10}$$

$$g_{\eta\tilde{\Sigma}^0_{ds}\tilde{\Sigma}^0_{ds}} = g_{\eta uu}F + g_{\eta ss}F + g_{\eta dd}(F-D) = -\sqrt{\frac{1}{6}}D, \tag{11}$$

and $(u \leftrightarrow s)$ to form two more auxiliary quantities

$$g_{\pi^0\tilde{\Sigma}^0_{us}\tilde{\Sigma}^0_{us}} = g_{\pi^0 dd}F + g_{\pi^0 ss}F + g_{\pi^0 uu}(F-D) = -\sqrt{\frac{1}{2}}D. \tag{12}$$

$$g_{\eta\tilde{\Sigma}^0_{us}\tilde{\Sigma}^0_{us}} = g_{\eta dd}F + g_{\eta ss}F + g_{\eta uu}(F-D) = -\sqrt{\frac{1}{6}}D. \tag{13}$$

The following relations hold:

$$2g_{\eta\tilde{\Sigma}^0_{ds}\tilde{\Sigma}^0_{ds}} + 2g_{\eta\tilde{\Sigma}^0_{us}\tilde{\Sigma}^0_{us}} - g_{\eta\Sigma^0\Sigma^0} = 3g_{\eta\Lambda\Lambda}. \tag{14}$$

$$g_{\pi\tilde{\Sigma}^0_{ds}\tilde{\Sigma}^0_{ds}} - g_{\pi\tilde{\Sigma}^0_{us}\tilde{\Sigma}^0_{us}} = \sqrt{3}g_{\pi\Sigma^0\Lambda}. \tag{15}$$

The origin of these relations lies in the structure of baryon wave functions in the NRQM with isospin $I = 1, 0$ and $I_3 = 0$:

$$2\sqrt{3}|\Sigma^0(ud,s)\rangle_\uparrow = |2u_\uparrow d_\uparrow s_\downarrow + 2d_\uparrow u_\uparrow s_\downarrow - u_\uparrow s_\uparrow d_\downarrow - s_\uparrow u_\uparrow d_\downarrow - d_\uparrow s_\uparrow u_\downarrow - s_\uparrow d_\uparrow u_\downarrow\rangle,$$

$$2|\Lambda\rangle_\uparrow = |d_\uparrow s_\uparrow u_\downarrow + s_\uparrow d_\uparrow u_\downarrow - u_\uparrow s_\uparrow d_\downarrow - s_\uparrow u_\uparrow d_\downarrow\rangle,$$

where $q_\uparrow$ ($q_\downarrow$) means wave function of the quark q (here $q = u, d, s$) with the helicity $+1/2$ (-1/2). With the exchanges $d \leftrightarrow s$ and $u \leftrightarrow s$ one arrives at the corresponding U-spin and V-spin quantities, so $U = 1, 0$ and $U_3 = 0$ baryon wave functions are

$$-2|\tilde{\Sigma}^0_{ds}(us,d)\rangle = |\Sigma^0(ud,s)\rangle + \sqrt{3}|\Lambda\rangle,$$

$$2|\tilde{\Lambda}_{ds}\rangle = -\sqrt{3}|\Sigma^0(ud,s)\rangle + |\Lambda\rangle,$$

while $V = 1, V_3 = 0$ and $V = 0$ baryon wave functions are

$$-2|\tilde{\Sigma}^0_{us}(ds,u)\rangle = |\Sigma^0(ud,s)\rangle - \sqrt{3}|\Lambda\rangle,$$

$$2|\tilde{\Lambda}_{us}\rangle = \sqrt{3}|\Sigma^0(ud,s)\rangle + |\Lambda\rangle.$$

It is easy to show that the relations given by the Eqs.(??, ??) immeaditely follow.

3 Relation between QCD correlators for Σ^0 and Λ hyperons

Now we show how similar considerations work for QCD sum rules on the example of QCD Borel sum rules for pseudoscalar meson-baryon coupling constants.

The starting point would be two-point Green's function for hyperons Σ^0 and Λ of the baryon octet:

$$\Pi^{\Sigma^0,\Lambda} = i\int d^4x e^{ipx} < 0|T\{J^{\Sigma^0,\Lambda}(x), J^{\Sigma^0,\Lambda}(0)\}|\eta >, \tag{16}$$

where isovector (with $I_3 = 0$) and isocalar field operators could be chosen as [?]

$$\begin{aligned} J^{\Sigma^0} &= \frac{1}{2}\epsilon_{abc}[(\mathrm{u^{aT}Cs^b})\gamma_5\mathrm{d^c} - (\mathrm{d^{aT}Cs^b})\gamma_5\mathrm{u^c} - \\ &\qquad (\mathrm{u^{aT}C\gamma_5 s^b})\mathrm{d^c} + (\mathrm{d^{aT}C\gamma_5 s^b})\mathrm{u^c}], \\ J^{\Lambda} &= \frac{1}{2\sqrt{3}}\epsilon_{abc}[-2(\mathrm{u^{aT}Cd^b})\gamma_5\mathrm{s^c} + (\mathrm{u^{aT}Cs^b})\gamma_5\mathrm{d^c} + (\mathrm{d^{aT}Cs^b})\gamma_5\mathrm{u^c} + \\ &\qquad 2(\mathrm{u^{aT}C\gamma_5 d^b})\mathrm{s^c} - (\mathrm{u^{aT}C\gamma_5 s^b})\mathrm{d^c} - (\mathrm{d^{aT}C\gamma_5 s^b})\mathrm{u^c}]. \end{aligned} \tag{17}$$

where a, b, c are the color indices and u, d, s are quark wave functions, C is charge conjugation matrix,

We show now that one can operate with with $\mathcal{M} = \pi^0, \eta$ - couplings to Σ hyperon and obtain the results for the $\pi\Lambda\Sigma$ and $\eta\Lambda\Lambda$ couplings. The reasoning would be valid also for charm and beaty Σ-like and Λ-like baryons.

In order to arrive at the desired relations we write not only isospin quantities but also U-spin and V-spin ones.

Let us introduce U-vector (with $U_3 = 0$) and U-scalar field operators just formally changing $(d \leftrightarrow s)$ in the Eq.(??):

$$\begin{aligned} J^{\tilde{\Sigma}^0_{ds}} &= \frac{1}{2}\epsilon_{abc}[(\mathrm{u^{aT}Cd^b})\gamma_5\mathrm{s^c} - (\mathrm{s^{aT}Cd^b})\gamma_5\mathrm{u^c} - \\ &\qquad (\mathrm{u^{aT}C\gamma_5 d^b})\mathrm{s^c} + (\mathrm{s^{aT}C\gamma_5 d^b})\mathrm{u^c}], \\ J^{\tilde{\Lambda}_{ds}} &= \frac{1}{2\sqrt{3}}\epsilon_{abc}[(-2(\mathrm{u^{aT}Cs^b})\gamma_5\mathrm{d^c} + (\mathrm{u^{aT}Cd^b})\gamma_5\mathrm{s^c} + \\ &\qquad (\mathrm{s^{aT}Cd^b})\gamma_5\mathrm{u^c}) \\ &\quad +2(\mathrm{u^{aT}C\gamma_5 s^b})\mathrm{d^c} - (\mathrm{u^{aT}C\gamma_5 d^b})\mathrm{s^c} - (\mathrm{s^{aT}C\gamma_5 d^b})\mathrm{u^c}]. \end{aligned} \tag{18}$$

Similarly we introduce V-vector (with $V_3 = 0$) and V-scalar field operators just changing $(u \leftrightarrow s)$ in the Eq.(??):

$$\begin{aligned} J^{\tilde{\Sigma}^0_{us}} &= \frac{1}{2}\epsilon_{abc}[(\mathrm{s^{aT}Cu^b})\gamma_5\mathrm{d^c} - (\mathrm{d^{aT}Cu^b})\gamma_5\mathrm{s^c} - \\ &\qquad (\mathrm{s^{aT}C\gamma_5 u^b})\mathrm{d^c} + (\mathrm{d^{aT}C\gamma_5 u^b})\mathrm{s^c}], \\ J^{\tilde{\Lambda}_{us}} &= \frac{1}{2\sqrt{3}}\epsilon_{abc}[(-2(\mathrm{s^{aT}Cd^b})\gamma_5\mathrm{u^c} + (\mathrm{s^{aT}Cu^b})\gamma_5\mathrm{d^c} + \\ &\qquad (\mathrm{d^{aT}Cu^b})\gamma_5\mathrm{s^c}) \\ &\quad +2(\mathrm{s^{aT}C\gamma_5 d^b})\mathrm{u^c} - (\mathrm{s^{aT}C\gamma_5 u^b})\mathrm{d^c} - (\mathrm{d^{aT}C\gamma_5 u^b})\mathrm{s^c}]. \end{aligned} \tag{19}$$

Field operators of the Eq.(??) and Eq.(??) can be related through

$$
\begin{aligned}
-\sqrt{3}J^{\tilde{\Lambda}_{ds}} &= J^{\Sigma^0} + \sqrt{3}J^{\Lambda}, \\
-2J^{\tilde{\Sigma}^0_{ds}} &= -J^{\Sigma^0} + \sqrt{3}J^{\Lambda}, \\
\sqrt{3}J^{\tilde{\Lambda}_{us}} &= -J^{\Sigma^0} + \sqrt{3}J^{\Lambda}, \\
2J^{\tilde{\Sigma}^0_{us}} &= J^{\Sigma^0} + \sqrt{3}J^{\Lambda}.
\end{aligned} \tag{20}
$$

Upon using Eqs.(??-??) two-point functions of the Eq.(??) for hyperons Σ^0 and Λ of the baryon octet can be related as

$$
2[\Pi^{\tilde{\Sigma}^0_{ds}} + \Pi^{\tilde{\Sigma}^0_{us}}] - \Pi^{\Sigma^0} = 3\Pi^{\Lambda}, \tag{21}
$$

$$
2[\Pi^{\tilde{\Lambda}_{ds}} + \Pi^{\tilde{\Lambda}_{us}}] - \Pi^{\Lambda} = 3\Pi^{\Sigma^0}. \tag{22}
$$

$$
2[\Pi^{\tilde{\Sigma}^0_{ds}} - \Pi^{\tilde{\Sigma}^0_{us}}] = \sqrt{3}[\Pi^{\Sigma^0\Lambda} + \Pi^{\Lambda\Sigma^0}], \tag{23}
$$

$$
2[\Pi^{\tilde{\Lambda}_{ds}} - \Pi^{\tilde{\Lambda}_{us}}] = -\sqrt{3}[\Pi^{\Sigma^0\Lambda} + \Pi^{\Lambda\Sigma^0}]. \tag{24}
$$

Relations given by Eqs.(??,??) enable us to write QCD sum rules for $\eta\Lambda\Lambda$ coupling starting from the one to Σ (*et vice versa*) while those given by Eqs.(??,??) allow to write QCD sum rules for the $\pi\Sigma\Lambda$- coupling.

4 Light cone sum rules for $g_{\pi\Lambda\Sigma}$ and $g_{\eta\Lambda\Lambda}$ couplings

We apply Eqs.(??-??) first to sum rules written for the pion and eta-coupling constants to octet baryons within the framework of the Light-Cone QCD [?]. As it was shown in [?] the corresponding LC QCD sum rules respected the unitary symmetry pattern. But sum rules for $\pi\Sigma\Lambda$- coupling and $\eta\Lambda\Lambda$ coupling were not elaborated. Let us write them now with the help of the Eqs.(??-??).

We rewrite LC QCD SR's (Eq.(40) from [?]) for the $\mathcal{M} = \pi, \eta$ coupling to the baryon $BB(qq, q')$, in a way to make clear unitary symmetry pattern:

$$
-\sqrt{\frac{1}{2}}m_B\lambda_B^2 g_{\mathcal{M}BB}e^{-(m_B^2/M^2)} = g_{\mathcal{M}qq}\Pi_1^{\gamma}(M^2) - g_{\mathcal{M}q'q'}\Pi_2^{\gamma}(M^2). \tag{25}
$$

wherefrom Eqs.(40) of [?] follows:

$$
\begin{aligned}
-m_N\lambda_N^2 g_{\pi NN}e^{-(m_N^2/M^2)} &= \Pi_1^{\gamma}(M^2) + \Pi_2^{\gamma}(M^2), \\
-m_\Sigma\lambda_\Sigma^2 g_{\pi^0\Sigma\Sigma}e^{-(m_\Sigma^2/M^2)} &= \Pi_1^{\gamma}(M^2), \\
-m_\Xi\lambda_\Xi^2 g_{\pi^0\Xi\Xi}e^{-(m_\Xi^2/M^2)} &= -\Pi_2^{\gamma}(M^2).
\end{aligned} \tag{26}
$$

and so on, where $\Pi_{1,2}^{\gamma}(M^2)$ are given in [?] and are rather cumbersome.

For us it is important to note that Π_1^{γ} corresponds exactly to $2F$, while (Π_2^{γ}) corresponds exactly to $(D - F)$ of the Eq.(??).

Thus up to a renormalization factor and some obvious redefinitions relations Eq.(??) and Eq.(??) are identical that is QCD sum rules shows explicit unitary symmetry pattern.

We can define $g_{\mathcal{M}\Sigma^0\Sigma^0}$ as follows

$$-\sqrt{\frac{1}{2}}m_{\Sigma^0}\lambda_{\Sigma^0}^2 g_{\mathcal{M}\Sigma^0\Sigma^0}e^{-(m_{\Sigma^0}^2/M^2)} =$$
$$\frac{1}{2}[g_{\mathcal{M}uu}\Pi_1^\gamma(M^2) + g_{\mathcal{M}dd}\Pi_1^\gamma(M^2)] - g_{\mathcal{M}ss}\Pi_2^\gamma(M^2). \tag{27}$$

Following reasoning of the previous section and analogues of the Eq.(??, ??) we obtain

$$-3\sqrt{\frac{1}{2}}m_\Lambda\lambda_\Lambda^2 g_{\mathcal{M}\Lambda\Lambda}e^{-(m_\Lambda^2/M^2)} =$$
$$(g_{\mathcal{M}uu} + g_{\mathcal{M}dd})(\frac{1}{2}\Pi_1^\gamma(M^2) - 2\Pi_2^\gamma(M^2))$$
$$+g_{\mathcal{M}ss}(2\Pi_1^\gamma(M^2) + \Pi_2^\gamma(M^2)) \tag{28}$$

$$-\sqrt{\frac{3}{2}}m_{\Lambda\Sigma^0}\lambda_\Lambda\lambda_{\Sigma^0} g_{\mathcal{M}\Lambda\Sigma^0}e^{-(m_{\Lambda\Sigma^0}^2/M^2)} =$$
$$(g_{\mathcal{M}uu} - g_{\mathcal{M}dd})(\Pi_1^\gamma(M^2) - 2\Pi_2^\gamma(M^2)) \tag{29}$$

$$m_{\Lambda\Sigma^0} = \frac{1}{2}(m_\Lambda + m_{\Sigma^0}) \tag{30}$$

But in [?] LC QCD SR's are flavour symmetric. So we search to investigate a more complicated case where unitary symmetry of v.e.v. and quark masses is broken.

5 QCD Sum Rules for $g_{\pi\Lambda\Sigma}$ and $g_{\eta\Lambda\Lambda}$ couplings

Recently Kim et al.[?]-[?] have elaborated QCD SR's for the π- and η- coupling constants to octet baryons taking into account corrections due to m_s and $\langle\bar{s}s\rangle$ so it would be ideal for us to take it as a probe and an independent test of our reasoning.

We shall prove our relations on the example of QCD Borel sum rules obtained in [?] (it corresponds in [?] to the choice of the Lorentz structure $i\gamma_5$ and the value of the parameter $t = -1$ which means the choice of Ioffe baryon current).

It is convenient for us to rewrite the result of [?] in common way only for $\mathcal{M} = \pi, \eta$ coupling to Σ^0 hyperon as

$$\frac{1}{\sqrt{2}}m_\mathcal{M}^2\lambda_{\Sigma^0}^2 g_{\mathcal{M}\Sigma^0\Sigma^0}e^{-(m_{\Sigma^0}^2/M^2)}[1 + A_{\Sigma^0}M^2] =$$
$$g_{\mathcal{M}ss}m_\mathcal{M}^2 M^4 E_0(x)[\frac{\langle\bar{s}s\rangle}{12\pi^2 f_\mathcal{M}} + \frac{3f_{3\mathcal{M}}}{4\sqrt{2}\pi^2}]$$
$$-g_{\mathcal{M}ss}\frac{1}{f_\mathcal{M}}M^2(m_d\langle\bar{u}u\rangle + m_u\langle\bar{d}d\rangle)\langle\bar{s}s\rangle$$
$$-g_{\mathcal{M}ss}\frac{m_\mathcal{M}^2}{72 f_\mathcal{M}}\langle\bar{s}s\rangle\langle\frac{\alpha_s}{\pi}\mathcal{G}^2\rangle$$
$$+\frac{1}{6f_\mathcal{M}}m_0^2[\langle\bar{s}s\rangle(m_d g_{\mathcal{M}uu}\langle\bar{u}u\rangle + m_u g_{\mathcal{M}dd}\langle\bar{d}d\rangle)$$
$$+m_s(g_{\mathcal{M}uu} + g_{\mathcal{M}dd})\langle\bar{u}u\rangle\langle\bar{d}d\rangle]. \tag{31}$$

(The corresponding formulae for any Σ-like baryon (p(uu,d), n(dd,u), $\Sigma^+(uu,s)$ etc) can be written from the Eq.(??) for the Σ^0 hyperon with appropriate changes of quark indices.)

Now we are able to construct two auxiliary sum rules for $g_{\mathcal{M}\Sigma^0_{ds}\Sigma^0_{ds}}$ and $g_{\mathcal{M}\Sigma^0_{us}\Sigma^0_{us}}$ upon changes $d \leftrightarrow s$ and $u \leftrightarrow s$ in the Eq.(??). So,

$$\begin{aligned}\frac{1}{\sqrt{2}} m^2_{\mathcal{M}} \lambda^2_{\Sigma^0_{ds}} g_{\mathcal{M}\Sigma^0_{ds}\Sigma^0_{ds}} e^{-(m^2_{\Sigma^0_{ds}}/M^2)}[1 + A_{\Sigma^0_{ds}} M^2] = \\ g_{\mathcal{M}dd} m^2_{\mathcal{M}} M^4 E_0(x)[\frac{\langle \bar{d}d\rangle}{12\pi^2 f_{\mathcal{M}}} + \frac{3 f_{3\mathcal{M}}}{4\sqrt{2}\pi^2}] \\ -g_{\mathcal{M}dd}\frac{1}{f_{\mathcal{M}}} M^2 (m_s \langle \bar{u}u\rangle + m_u \langle \bar{s}s\rangle)\langle \bar{d}d\rangle \\ -g_{\mathcal{M}dd}\frac{m^2_{\mathcal{M}}}{72 f_{\mathcal{M}}}\langle \bar{d}d\rangle\langle \frac{\alpha_s}{\pi}\mathcal{G}^2\rangle \\ +\frac{1}{6 f_{\mathcal{M}}} m^2_0 [\langle \bar{d}d\rangle(m_s g_{\mathcal{M}uu}\langle \bar{u}u\rangle + m_u g_{\mathcal{M}ss}\langle \bar{s}s\rangle) \\ +m_d (g_{\mathcal{M}uu} + g_{\mathcal{M}ss})\langle \bar{u}u\rangle\langle \bar{s}s\rangle].\end{aligned} \tag{32}$$

$$\begin{aligned}\frac{1}{\sqrt{2}} m^2_{\mathcal{M}} \lambda^2_{\Sigma^0_{us}} g_{\mathcal{M}\Sigma^0_{us}\Sigma^0_{us}} e^{-(m^2_{\Sigma^0_{us}}/M^2)}[1 + A_{\Sigma^0_{us}} M^2] = \\ g_{\mathcal{M}uu} m^2_{\mathcal{M}} M^4 E_0(x)[\frac{\langle \bar{u}u\rangle}{12\pi^2 f_{\mathcal{M}}} + \frac{3 f_{3\mathcal{M}}}{4\sqrt{2}\pi^2}] \\ -g_{\mathcal{M}uu}\frac{1}{f_{\mathcal{M}}} M^2 (m_d \langle \bar{s}s\rangle + m_s \langle \bar{d}d\rangle)\langle \bar{u}u\rangle \\ -g_{\mathcal{M}uu}\frac{m^2_{\mathcal{M}}}{72 f_{\mathcal{M}}}\langle \bar{u}u\rangle\langle \frac{\alpha_s}{\pi}\mathcal{G}^2\rangle \\ +\frac{1}{6 f_{\mathcal{M}}} m^2_0 [\langle \bar{u}u\rangle(m_d g_{\mathcal{M}ss}\langle \bar{s}s\rangle + m_s g_{\mathcal{M}dd}\langle \bar{d}d\rangle) \\ +m_u (g_{\mathcal{M}ss} + g_{\mathcal{M}dd})\langle \bar{s}s\rangle\langle \bar{d}d\rangle].\end{aligned} \tag{33}$$

Now we can construct sum rule for $\pi\Lambda\Sigma$ and $\eta\Lambda\Lambda$ couplings. Upon using the Eqs. (??,??,??) we get

$$\begin{aligned}\sqrt{\frac{3}{2}} m^2_{\mathcal{M}} \lambda_{\Lambda}\lambda_{\Sigma^0} g_{\mathcal{M}\Sigma^0\Lambda} e^{-(m^2_{\Sigma^0\Lambda}/M^2)}[1 + A_{\Sigma^0\Lambda} M^2] = \\ m^2_{\mathcal{M}} M^4 E_0(x)[\frac{g_{\mathcal{M}dd}\langle \bar{d}d\rangle - g_{\mathcal{M}uu}\langle \bar{u}u\rangle}{12\pi^2 f_{\mathcal{M}}}] \\ -\frac{1}{f_{\mathcal{M}}} M^2 [(m_s\langle \bar{u}u\rangle + m_u\langle \bar{s}s\rangle)\langle \bar{d}d\rangle g_{\mathcal{M}dd} \\ -(m_d\langle \bar{s}s\rangle + m_s\langle \bar{d}d\rangle)\langle \bar{u}u\rangle g_{\mathcal{M}uu}] \\ -\frac{m^2_{\mathcal{M}}}{72 f_{\mathcal{M}}}[\langle \bar{d}d\rangle g_{\mathcal{M}dd} - \langle \bar{u}u\rangle g_{\mathcal{M}uu}]\langle \frac{\alpha_s}{\pi}\mathcal{G}^2\rangle \\ +\frac{1}{6 f_{\mathcal{M}}} m^2_0 [m_s \langle \bar{d}d\rangle\langle \bar{u}u\rangle(g_{\mathcal{M}uu} - g_{\mathcal{M}dd}) \\ -m_u\langle \bar{d}d\rangle\langle \bar{s}s\rangle g_{\mathcal{M}dd} + m_d\langle \bar{u}u\rangle\langle \bar{s}s\rangle g_{\mathcal{M}uu}].\end{aligned} \tag{34}$$

Finally for $g_{\eta\Lambda\Lambda}$ and $g_{\pi\Sigma\Lambda}$.

$$\sqrt{3}m_\eta^2\lambda_\Lambda^2 g_{\eta\Lambda\Lambda}e^{-(M^2/m_\Lambda^2)}[1+A_\Lambda M^2] =$$
$$\frac{1}{3}m_\eta^2 M^4 E_0(x)[\frac{\langle\bar{u}u\rangle+\langle\bar{d}d\rangle+\langle\bar{s}s\rangle}{6\pi^2 f_\eta}+\frac{9\sqrt{3}f_{3\eta}}{4\pi^2}]$$
$$-\frac{4M^2}{3f_\eta}[(m_d\langle\bar{u}u\rangle+m_u\langle\bar{d}d\rangle)\langle\bar{s}s\rangle+m_s\langle\bar{u}u\rangle\langle\bar{d}d\rangle]$$
$$-\frac{m_\eta^2}{108 f_\eta}[\langle\bar{u}u\rangle+\langle\bar{d}d\rangle+\langle\bar{s}s\rangle] < \frac{\alpha_s}{\pi}\mathcal{G}^2 >$$
$$+\frac{m_0^2}{18 f_\eta}[-7(m_d\langle\bar{u}u\rangle+m_u\langle\bar{d}d\rangle)\langle\bar{s}s\rangle+2m_s\langle\bar{u}u\rangle\langle\bar{d}d\rangle]. \qquad (35)$$

$$\sqrt{3}m_\pi^2\lambda_\Lambda\lambda_{\Sigma^0}g_{\pi\Sigma\Lambda}e^{-(m_{\Sigma\Lambda}^2/M^2)}[1+A_{\Sigma\Lambda}M^2] =$$
$$-m_\pi^2 M^4 E_0(x)[\frac{\langle\bar{d}d\rangle+\langle\bar{u}u\rangle}{12\pi^2 f_\pi}]$$
$$+\frac{1}{f_\pi}M^2[(m_s\langle\bar{u}u\rangle+m_u\langle\bar{s}s\rangle)\langle\bar{d}d\rangle+(m_d\langle\bar{s}s\rangle+m_s\langle\bar{d}d\rangle)\langle\bar{u}u\rangle]$$
$$+\frac{m_\pi^2}{72 f_\pi}[\langle\bar{d}d\rangle+\langle\bar{u}u\rangle]\langle\frac{\alpha_s}{\pi}\mathcal{G}^2\rangle$$
$$+\frac{1}{6 f_\pi}m_0^2[m_s\langle\bar{d}d\rangle\langle\bar{u}u\rangle 2+m_u\langle\bar{d}d\rangle\langle\bar{s}s\rangle+m_d\langle\bar{u}u\rangle\langle\bar{s}s\rangle]. \qquad (36)$$

It is straightforward to show that starting from the Eq.(??) and applying Eq.(??) one returns to the Eq.(??).

6 Numerical analysis and discussion

In this section we analyze the sum rule for coupling constant $g_{\pi\Sigma\Lambda}$ obtained in the previous section. Numerical results are given in the table 1,2 and in the Figs 1-3. We put our QCD SR's results for the baryon octet in the columns 'QCD' and 'Zero' of the Table 2.

As in [?] we use $\lambda_B = C \cdot M_B^6$, where M_B is the baryon mass, while C is a universal constant and is derived from the known coupling constant $g_{\pi NN} = 13.4$.

The values of other input parametrs appearing in the sum rules are: $f_\eta = f_\pi = 93MeV$, $f_{3\eta} = f_{3\pi} = 0$, $m_0^2 = 0.8GeV^2$, $\langle\frac{\alpha_s}{\pi}\mathcal{G}^2\rangle = 0.33^4 GeV^4$, $\langle\bar{u}u\rangle = \langle\bar{d}d\rangle = \langle\bar{q}q\rangle = -(0.23)^3 GeV^3, \langle\bar{s}s\rangle = 0.8\langle\bar{q}q\rangle$, $S = -2.07$, $m_\eta = 548MeV, m_\pi = 140MeV$, $m_u = m_d = 5MeV, m_s = 150MeV$.

Note that we use $g_{\pi NN} = g_{\pi^+ np} = 13.4$, that is, $g_{\pi NN}^2/4\pi = 14.3$. It seems to us that in [?] $g_{\pi NN} = g_{\pi^0 pp} = 13.4$ was used which resulted in $g_{\pi^+ np} = 13.4 \cdot \sqrt{2} = 18.95$ and $g_{\pi^+ np}^2/4\pi = 28.6$. We put all SU(3) results into the Table 1.

Table 2 shows the results of QCD SR's calculations. It is seen that coupling constants $g_{\pi\Sigma^0\Lambda}$ and $g_{\pi\Xi^0\Xi^0}$ are too large. As the terms of the kind

$$-g_{\mathcal{M}ss}\frac{1}{f_\eta}M^2(m_d\langle\bar{u}u\rangle+m_u\langle\bar{d}d\rangle)\langle\bar{s}s\rangle \qquad (37)$$

in the Eq. (??) give anomaly large contribution into these couplings resulting in huge coupling constants (see 3th column of the Table 2), we put also the results with subtraction of these terms (4th column of the Table 2). One can see that the values with subtraction look more reasonable and let to us a hope that the higher contributions should cancel anomaly.

The main result of our work is the derivation of new relations between QCD Borel sum rules for strong coupling constants. It is shown that starting from the sum rule for the coupling constant $g_{\mathcal{M}\Sigma\Sigma}$, $\mathcal{M} = \pi, \eta$, it is straightforward to obtain the corresponding sum rules for the $g_{\pi\Lambda\Sigma}$ and $g_{\eta\Lambda\Lambda}$ couplings. The values of the constants obtained show large symmetry breaking and can be hardly regarded literally for a moment. The overall sum rule pattern shows characteristic symmetry structure and deserves further study.

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Table 1. In the 1st column there is SU(3) result given by [?]; in the 2nd column there is our reproduction of this result with F/D=0.213 and $g_{\pi^+pn} = F + D = 13.4 \cdot \sqrt{2}$; in the 3rd column there is SU(3) result with F/D=0.213 and $g_{\pi^+pn} = F + D = 13.4$; in the 4th column there is SU(3) result with F/D=2/3 and $g_{\pi^+pn} = F + D = 13.4$;

	[?]	Reproduction	Correct	Standart
F/D	~0.2	0.213	0.213	2/3
$g_{\pi pn}$	?	18.95	13.4	13.4
$g_{\pi NN}$	13.4	–	–	–
$g_{\pi pp}$	?	13.4	9.48	9.48
$g_{\eta pp}$	-2.3	-2.30	-1.63	3.28
$g_{\pi\Sigma^+\Sigma^+}$	4.7	4.71	3.33	7.58
$g_{\eta\Sigma^+\Sigma^+}$	12.8	12.76	9.02	6.56
$g_{\pi\Xi^0\Xi^0}$	-8.7	8.69	6.15	1.90
$g_{\eta\Xi^0\Xi^0}$	-10.5	-10.45	-7.39	-9.85
$g_{\eta\Sigma^0\Sigma^0}$	-	12.76	9.02	6.56
$g_{\eta\Lambda\Lambda}$	-	-12.76	-9.02	-6.56
$g_{\pi\Sigma^0\Lambda}$	-	12.76	9.02	6.56

Table 2 In the 1th column there is QCD SR result obtained in [?]; in the 2th column there is our QCD SR result obtained along [?] with $g_{\pi^+ pn} = F + D = 13.4 \cdot \sqrt{2}$ as input and our results for $\pi\Sigma\Lambda$ and $\eta\Lambda\Lambda$; in the 3th column there is our QCD SR result obtained along [?] but with $g_{\pi^+ pn} = F + D = 13.4$ as input and our results for $\pi\Sigma\Lambda$ and $\eta\Lambda\Lambda$; in the 4th column there is the same results as in the 3th one but without contiribution of ?? term.

Константа	[?]	This work	QCD	Zero
$g_{\pi pp}(input)$	13.4	13.4	9.48	9.48
$g_{\eta pp}$	-0.63	-1.01	-0.71	-1.47
$g_{\pi\Sigma^+\Sigma^+}$	14.1	16.74	11.84	29.83
$g_{\eta\Sigma^+\Sigma^+}$	1.4	2.44	1.72	4.03
$g_{\pi\Xi^0\Xi^0}$	-38.7	-47.77	-33.8	-2.81
$g_{\eta\Xi^0\Xi^0}$	-2.3	-3.20	-2.26	-2.61
$g_{\eta\Sigma^0\Sigma^0}$	1.4	2.42	1.71	4.01
$g_{\eta\Lambda\Lambda}$	–	-4.68	-3.31	-3.07
$g_{\pi\Sigma^0\Lambda}$	–	71.64	-50.66	-24.43

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Алтуг Озпинечи
Станислав Борисович Яковлев
Валерий Семенович Замиралов

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117246, Москва, ул. Обручева,55А
Тел./факс(095)718-6966, 718-7776, 718-7785
e-mail:izdat@abiturcenter.ru
http:/www.abiturcenter.ru

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